$$\begin{cases} I + J = \dfrac{e^{\pi} - 1}{2} (1) \\ I - J = -\dfrac{1}{4}\left(1 + e^{\pi}\right) (2) \end{cases}$$

$$(1)+(2) \Rightarrow 2I = \frac{e^{\pi}-1}{2} - \frac{1+e^{\pi}}{4}$$

$$= \frac{2e^{\pi} - 2 - 1 - e^{\pi}}{4} = \frac{e^{\pi}-3}{4}$$

D'où $\boxed{I = \dfrac{e^{\pi}-3}{8}}$

$$(1) \Rightarrow J = \frac{e^{\pi}-1}{2} - I = \frac{e^{\pi}-1}{2} - \left(\frac{e^{\pi}-3}{8}\right)$$

$$= \frac{4e^{\pi} - 4 - e^{\pi} + 3}{8} = \frac{3e^{\pi}-1}{8} \quad \boxed{J = \frac{3e^{\pi}-1}{8}}$$

Conclusion :

$$I = \frac{e^{\pi}-3}{8} \quad \text{et} \quad J = \frac{3e^{\pi}-1}{8}.$$

Exercice 7

1) $\forall x \in \mathbb{R}^*$

$$1 + \frac{1}{e^x - 1} = \frac{e^x - 1}{e^x - 1} + \frac{1}{e^x - 1}$$

$$= \frac{e^x - 1 + 1}{e^x - 1} = \frac{e^x}{e^x - 1}$$

Donc $\forall x \in \mathbb{R}^*$: $\boxed{\dfrac{e^x}{e^x - 1} = 1 + \dfrac{1}{e^x - 1}}$

2) a) $I = \displaystyle\int_{\ln 2}^{\ln 3} \frac{e^t}{e^t - 1} dt = \int_{\ln 2}^{\ln 3} \frac{\left(e^t - 1\right)'}{e^t - 1} dt$

$$= \left[\ln\left(e^t - 1\right)\right]_{\ln 2}^{\ln 3}$$

$$= \ln\left(e^{\ln 3} - 1\right) - \ln\left(e^{\ln 2} - 1\right)$$

$$= \ln(3-1) - \ln(2-1)$$

$$= \ln 2 - \ln 1 = \ln 2$$

Donc: $\boxed{I = \ln 2}$

b) $J = \displaystyle\int_{\ln 2}^{\ln 3} \frac{1}{e^t - 1} dt$

On sait que: $\forall t \in \mathbb{R}^* \ \dfrac{e^t}{e^t - 1} = 1 + \dfrac{1}{e^t - 1}$

D'où $\dfrac{1}{e^t - 1} = \dfrac{e^t}{e^t - 1} - 1$

Par suite:

$$J = \int_{\ln 2}^{\ln 3} \left(\frac{e^t}{e^t - 1} - 1\right) dt = \int_{\ln 2}^{\ln 3} \frac{e^t}{e^t - 1} dt - \int_{\ln 2}^{\ln 3} dt$$

$$= I - \left[t\right]_{\ln 2}^{\ln 3} = \ln 2 - [\ln 3 - \ln 2]$$

$$= \ln 2 - \ln 3 + \ln 2 = 2\ln 2 - \ln 3$$

Donc : $\boxed{J = 2\ln 2 - \ln 3}$

3) $K = \displaystyle\int_{\ln 2}^{\ln 3} \frac{e^{2t}}{e^t - 1} dt$

$$K - I = \int_{\ln 2}^{\ln 3} \frac{e^{2t}}{e^t - 1} dt - \int_{\ln 2}^{\ln 3} \frac{e^t}{e^t - 1} dt$$

$$= \int_{\ln 2}^{\ln 3} \left(\frac{e^{2t}}{e^t - 1} - \frac{e^t}{e^t - 1}\right) dt$$

$$= \int_{\ln 2}^{\ln 3} \frac{e^{2t} - e^t}{e^t - 1} dt = \int_{\ln 2}^{\ln 3} \frac{e^t\left(e^t - 1\right)}{e^t - 1} dt$$

$$= \int_{\ln 2}^{\ln 3} e^t dt = \left[e^t\right]_{\ln 2}^{\ln 3} = e^{\ln 3} - e^{\ln 2} = 3 - 2 = 1$$

Par suite: $K - I = 1$, il en résulte: $K = I + 1$

D'où : $\boxed{K = \ln 2 + 1}$

Exercice 8

I/ 1) $g(x) = (1-x)e^{-x} + 1, x \in \mathbb{R}$

$$g'(x) = (1-x)'e^{-x} + \left(e^{-x}\right)'(1-x) + 0$$

$$= -e^{-x}(1 + 1 - x) = -e^{-x}(2 - x)$$

$$= (x-2)e^{-x}, e^{-x} > 0$$

X	$-\infty$		2		$+\infty$
$g'(x)$		$-$	0	$+$	
$g(x)$		↘	$1-e^{-2}$	↗	